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Solution by Artemas Martin, LL.D., Washington, D. C.

Let x^2, y^2, z^2 be three square numbers in arithmetical progression; then we must have

$$y^2 - x^2 = z^2 - y^2, \qquad \text{or} \qquad x^2 + z^2 = 2y^2. \tag{1}$$

Assume $z = v + w$, $x = v - w$, and (1) becomes after dividing by 2,

$$v^2 + w^2 = y^2. \tag{2}$$

Take now $v = p^2 - q^2$, $w = 2pq$, and (2) is satisfied. Retracing, we find

$$z = p^2 - q^2 + 2pq, \qquad x = p^2 - q^2 - 2pq, \qquad y = p^2 + q^2.$$

Hence, the required squares are

$$x^2 = (p^2 - q^2 - 2pq)^2, \qquad y^2 = (p^2 + q^2)^2, \qquad z^2 = (p^2 - q^2 + 2pq)^2.$$

Taking $p = 2$, $q = 1$, the numbers are 1, 25, 49; taking $p = 3$, $q = 2$, the numbers are 49, 169, 289; taking $p = 4$, $q = 1$, the numbers are 49, 289, 529; taking $p = 4$, $q = 3$, the numbers are 289, 625, 961; and so on, indefinitely.

The common difference of three square numbers in arithmetical progression can not be a square number. See Barlow's "Theory of Numbers," p. 257. An equivalent theorem is also given in Carmichael's *Diophantine Analysis*, p. 14.

There can not be *four* square numbers in arithmetical progression. Barlow, same page. Therefore there can not be *five*, nor any greater number than three, of squares in arithmetical progression.

Also solved by J. L. Riley, V. M. Spunar, and the Proposer.

265 (Number Theory). Proposed by J. W. NICHOLSON, Louisiana State University.

If the roots of $x^4 - ax^2 + bx + c = 0$ are rational, prove that $4(a + yz) - 3(y + z)^2$ is a perfect square, y and z being any two roots of the equation.

Solution by N. P. Pandya, Sojitra, India.

Since y and z are roots of the given equation, $x^2 - x(y + z) + yz$ is a factor of the left-hand side of the equation.

Since the term in x^3 is wanting, the remaining roots are given by

$$x^2 + x(y + z) + \frac{c}{yz} = 0. \tag{1}$$

The product of (1) with $x^2 - x(y + z) + yz = 0$ gives

$$a = -\frac{c}{yz} - yz + (y + z)^2.$$

Hence,

$$4(a + yz) = -\frac{4c}{yz} + 4(y + z)^2,$$

or

$$4(a + yz) - 3(y + z)^2 = (y + z)^2 - \frac{4c}{yz} = \text{a square},$$

since the roots of (1) are rational and its discriminant is therefore a square.

267 (Number Theory). Proposed by C. C. YEN, Tangshan, North China.

A number theory function $\phi(n)$ is defined for every positive integer n, and for every such number n it satisfies the relation $\phi(d_1) + \phi(d_2) + \phi(d_3) + \cdots + \phi(d_r) = n$, where $d_1, d_2, \cdots, d_r$ are the divisors of n. From this property alone show that

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_k}\right),$$

where $p \quad p \quad p_3 \cdots p_k$ are the different prime factors of n.